Quadratic and Linear Functions

1. Convert each of the following into the form $y = a(x-h)^2 + k$

a) $y = 3x^2 + 2x + 1$

$$y - 1 = 3\left(x^2 + \frac{2}{3}x\right)$$
$$y - 1 + \frac{4}{12} = 3\left(x^2 + \frac{2}{3}x + \frac{4}{36}\right)$$
$$y + \frac{(-1)\cdot 12 + 4}{12} = 3\left(x + \frac{4}{6}\right)^2$$
$$y - \frac{8}{12} = 3\left(x + \frac{4}{6}\right)^2$$
$$y = 3\left(x + \frac{4}{6}\right)^2 + \frac{8}{12}$$

b) $y = -2x^2 + 2x - 1$

$$y + 1 = -2\left(x^2 - \frac{2}{2}x\right)$$
$$y + 1 - \frac{4}{8} = -2\left(x^2 - \frac{2}{2}x + \frac{4}{16}\right)$$
$$y + \frac{(1)\cdot 8 - 4}{8} = -2\left(x - \frac{2}{4}\right)^2$$
$$y + \frac{4}{8} = -2\left(x - \frac{2}{4}\right)^2$$
$$y = -2\left(x - \frac{2}{4}\right)^2 - \frac{4}{8}$$

2. For each of the given questions determine the indicated information:

a) $y = -3(x+2)^2 + 1$

1.	direction of opening	down
2.	shape	narrow
3.	max/min point	max
4.	max/min value	y = 1
5.	axis of symmetry	x = -2
6.	coordinates of vertex	(-2, 1)
7.	y-intercept	-11
8.	domain of function	$x \in R$
9.	range of function	$y \le 1$

10. Table of Values

x	5	-4	-2	0	1
y	-26	-11	1	-11	-26

b) $y = 4(x-3)^2 - 2$

1.	Direction of opening	up
2.	Shape	narrow
3.	max/min point	min
4.	Max/min value	y = -2
5.	Axis of symmetry	x = 3
6.	Coordinates of vertex	(3, 2)
7.	y-intercept	34
8.	Domain of function	$x \in R$
9.	Range of function	$y \ge -2$

10. Table of Values

x	-1	0	3	6	7
y	62	34	2	34	62

3. Using the appropriate formula determine the equation of the axis of symmetry, the min/max value and the coordinated of the vertex of the equation $y = 2x^2 - 6x - 5$

$$x = -\frac{b}{2a}$$
$$x = -\frac{(-6)}{2(2)}$$
$$x = \frac{6}{4}$$

axis of symmetry

$$y = \frac{4ac - b^2}{4a}$$
$$y = \frac{4(2)(-5) - (-6)^2}{4(2)}$$
$$y = \frac{-40 - 36}{8} = -\frac{76}{8}$$

min/max value

$$\left(\frac{6}{4}, -\frac{76}{8}\right)$$

vertex

4. Determine the required info:
   a) slope and y-intercept of the equation: 5x - 8y = -12

$$-8y = -5x - 12$$
$$y = \frac{-5}{-8}x - \frac{12}{-8}$$
$$m = \frac{5}{8}, b = \frac{12}{8}$$

   b) slope, midpoint, distance of the line joining the points (-5, -9) and (-3, 11)

$$m = \frac{y_2 - y_2}{x_2 - x_1}$$
$$m = \frac{-9 - 11}{-5 - (-3)}$$
$$m = \frac{-20}{-2} = \frac{20}{2} = 10$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$M\left(\frac{-5 + (-3)}{2}, \frac{-9 + 11}{2}\right)$$
$$M\left(\frac{-8}{2}, \frac{2}{2}\right) = M(-4, 1)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{((-5 - (-3))^2 + (-9 - 11)^2}$$
$$d = \sqrt{(-2)^2 + (-20)^2} = \sqrt{404}$$

5. Determine the equation of the line given: ***(NO FRACTIONAL COEFFICIENTS)***

   a) m = 2/3 and b = -6

$$y = mx + b \Rightarrow y = \frac{2}{3}x + (-6) \Rightarrow 3(y) = 3 \cdot \frac{2}{3}x + (-6) \cdot 3 \Rightarrow 3y = 2x - 18$$

   b) m = -4 and contains the point (-2, -4)

$$(y_2 - y_1) = m(x_2 - x_1)$$
$$(y - (-4)) = -4(x - (-2))$$
$$y + 4 = -4x - 8$$
$$y = -4x - 12$$

   c) passes through the points (2, 7) and (-4, -3)

$$m=\frac{y_2-y_2}{x_2-x_1}=\frac{7-(-3)}{2-(-4)}=\frac{10}{6}=\frac{5}{3}$$

$$(y_2-y_1)=m(x_2-x_1)\Rightarrow(y-7)=\frac{5}{3}(x-2)\Rightarrow 3\cdot(y-7)=3\cdot\frac{5}{3}(x-2)\Rightarrow 3y-21=5x-10$$

$$3y=5x+11$$

d) passes through the point (1,3) and is parallel to a line with equation 5x -3y = 4

$$-3y=-5x+4\Rightarrow y=\frac{-5}{-3}x+\frac{4}{-3}\Rightarrow m_1=\frac{5}{3},\Rightarrow m_2=\frac{5}{3}$$

$$(y_2-y_1)=m(x_2-x_1)\Rightarrow(y-3)=\frac{5}{3}(x-1)\Rightarrow 3\cdot(y-3)=3\cdot\frac{5}{3}(x-1)\Rightarrow$$

$$3y-3=5x-5\Rightarrow 3y=5x-2$$

e) of the perpendicular bisector of the line segment joining the points (6, 2) and (10, -4)

$$m=\frac{y_2-y_2}{x_2-x_1}=\frac{2-(-4)}{6-10}=\frac{6}{-4}=-\frac{6}{4}$$

$$m_1=-\frac{6}{4},\therefore m_2=\frac{4}{6}$$

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{6+10}{2},\frac{2+(-4)}{2}\right)=\left(\frac{16}{2},\frac{-2}{2}\right)=(8,-1)$$

$$(y_2-y_1)=m(x_2-x_1)\Rightarrow(y-(-1))=\frac{4}{6}(x-8)\Rightarrow 6\cdot(y-(-1))=6\cdot\frac{4}{6}(x-8)\Rightarrow$$

$$6y+6=4x-32\Rightarrow 6y=4x-38$$